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Compact analytical model for overall thermal resistance of bolted joints

M. B. H. MANTELLI†

Satellite Thermal Control Group, Department of Mechanical Engineering, CP 476, Federal University of Santa Catarina, Florianopolis, SC 88010-970, Brazil

and

M. M. YOVANOVICH

Microelectronics Heat Transfer Laboratory, Department of Mechanical Engineering, University of Waterloo, Waterloo, Ontario, Canada N2L 3G1

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Abstract—A compact model to determine the overall thermal resistance of bolted joints, which considers a variable number of washers between plates, is presented. Conduction models are developed for material and constriction resistances. The contact model is obtained from the literature. The temperature influence is studied using thermal stress analysis. Non-important resistances are eliminated from the resulting resistance network and algebraic simplifications are applied. The resulting compact model is compared to the literature models, correlations and data for the no washer case. The author's data, for the one and three washer between plate configurations, are also compared with the model. © 1998 Elsevier Science Ltd.

INTRODUCTION

The importance of the thermal contact resistance in many heat transfer mechanisms is well recognized, especially for the thermal control of electronic equipment in space applications. In the last decades, a great effort has been spent in the theoretical prediction of the thermal contact resistance and in the generation of data to be compared with the models and/or to be used in the development of correlations. However, in complex systems, such as bolted joints, other thermal resistances are found, including material, constriction, radiation, etc. In the current literature, most of the thermal studies of bolted joints are concentrated only in the contact resistance between plates fastened by a bolt. The constriction resistances of plates which are subject to specific boundary conditions, are less often studied. Other heat transfer paths, such as the bolt path, are simply neglected without having its value estimated. These approximations can lead to great errors, since, under certain conditions, the importance of these resistances can increase. The literature models also do not consider the presence of washers between plates.

The main objective of this work is to identify the thermal resistances found in bolted joints, to connect them in a network and to obtain an overall thermal resistance model through algebraic simplifications. The study is based on a satellite bolted joint, but the resulting model can be applied to other configurations.

Typical satellite bolted joints, such as those designed for the First Brazilian Data Collection Satellite (*SCD*1), are composed of two thin plates of similar thicknesses connected by a bolt, as shown in the schematic in the top of Fig. 1. Identical washers are positioned between the plates, between the bolt head and the upper plate and between the nut and the lower plate. Heat is supplied to the upper plate by radiation from the satellite internal environment. It can be also supplied by conduction, when, for instance, a box containing electronic equipment is connected to one of the plates. Most of the heat dissipated from the lower plate is by radiation. A vacuum environment is assumed, therefore convection heat transfer is considered negligible.

LITERATURE REVIEW

Until recently, the thermal studies in bolted joints were essentially experimental. Aron and Colombo [1], in 1963, Elliot [2], in 1965, and Fontenot [3], in 1968, were among the first researchers who studied the thermal aspects of bolted joints. The experimental data generated by these researchers were used in the com-

† Presently a visiting researcher from Brazilian Institute for Space Research (INPE), São José dos Campos, SP, Brazil.

NOMENCLATURE

a internal washer and plate radius [m]
A heat flow area [m^2]
b external washer radius [m]
Bi Biot number ($h_c t/k$)
c external plate radius [m]
C_1, C_2, C_3, C_4 integration constants
E modulus of elasticity [N/m^2]
h heat transfer coefficient [W/m^2 K]
H hardness of the softer contacting material [N/m^2]
$I_{0,1}$ modified Bessel functions of first (0) and second (1) kinds
k conductivity [W/m K]
$K_{0,1}$ modified Bessel functions of first (0) and second (1) kinds
L length of material under stress [m]
m Bessel function argument ($m^2 = h_c/(k_p L_p)$), asperity slope
n number of washers
P stress [N/m^2]
q heat flux per unit area [W/m^2]
Q heat input [W]
r radius [m]
r_b bolt shaft radius [m]
R thermal resistance [K/W]
R^* non-dimensional resistance ($R^* = Rk_sL_s$, where $k_s = 2k_pk_w/k_p+k_w$)
t thickness [m]
T temperature [K]
T^* non-dimensional temperature ($T^* = T_mk_sL_s/qb^2$)
$\bar{T}$ mean temperature [K]
w displacement [m].

Greek symbols
α coefficient of thermal expansion [m/m/K]
Δ difference
ε emissivity
θ inner ring temperature ($\theta = T-T_s$) [K]
λ Φ parameter argument
σ Stefan–Boltzmann constant [W/m^2 K^4], RMS roughness
Φ non-dimensional contact parameter
ψ outer ring temperature ($\psi = T-T_s$) [K].

Subscripts
al aluminum
b bolt
c contact
cbn contact, bolt–nut
ct constriction
f final
h bolt head
i initial
n nut
m material, mean
mb material, bolt shaft
p plate
p_1 upper plate
p_2 lower plate
r radial, radiation
s surface, harmonic mean
ss stainless steel
w washer
w_0 washer between bolt head and upper plate
w_1 to w_n washers between plates
w_{n+1} washer between lower plate and nut
0 ambient
1 contact surface 1
2 contact surface 2.

parison with model of Mittlebach *et al.* [4], in 1992, where the results compared favorably with the thermal conductance data for two aluminum plates fastened by a bolt.

In 1990, Fletcher *et al.* [5] studied the macroscopic resistance in bolted joints. They neglected the thermal contact resistance and conducted experiments to determine the constriction resistance of plates connected by a bolt, using the electrolytic analog technique. The heat was considered applied uniformly to one of the surfaces of the plate and then constricted to the plate contact area. All the other external surfaces of the plates were considered insulated. They compared the experimental data with the results obtained numerically using the finite element method (FEM). The maximum error was about 8%. Based on the analysis and experimental data, they developed a correlation relating non-dimensional macroscopic thermal resistance to the geometric variables, which compared with experimental data (electrolytic analog technique) within 15%. The variables considered included the upper and lower plate thicknesses, the plate radius, the contact zone radius and the bolt radius. Lee *et al.* [6], in 1993, developed an analytical model for the physical model studied experimentally by Fletcher *et al.* [5]. The comparison of the analytical with the experimental results was good. They obtained another correlation based on the following par-

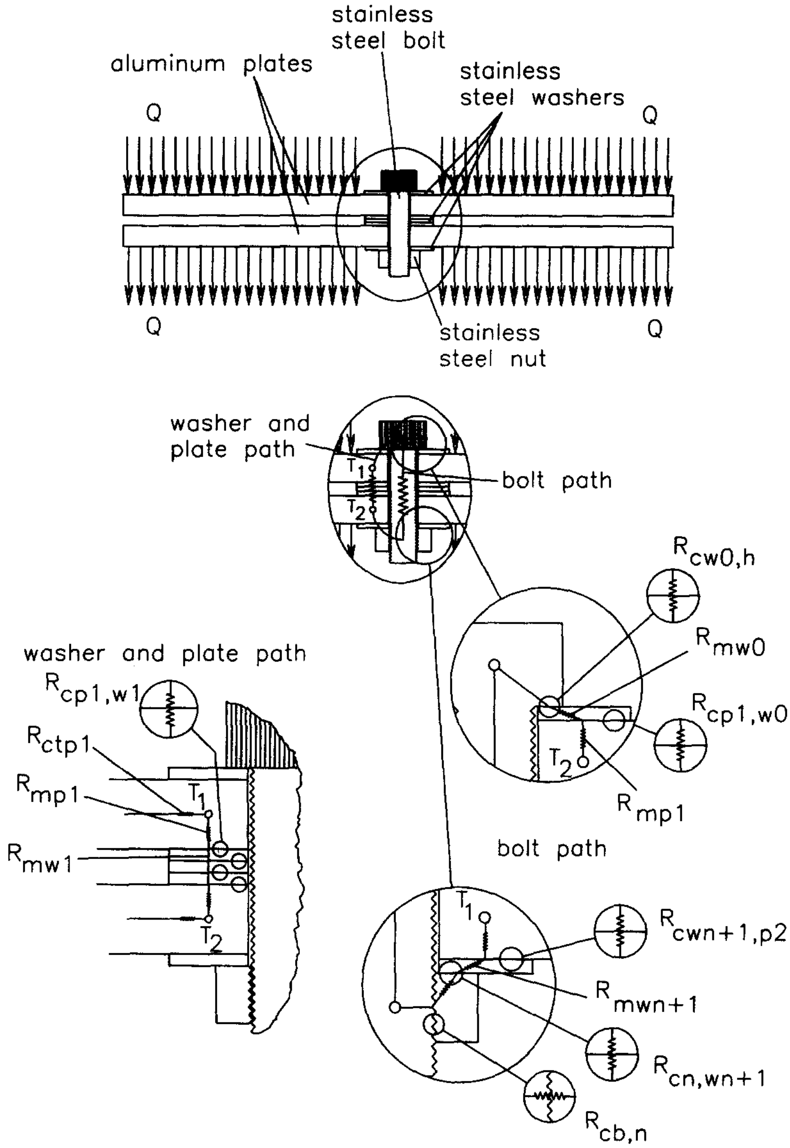

Fig. 1. Thermal resistances and resistance paths for a typical satellite bolted joint.

ameters: bolt-hole radius, plate radius, plate thickness, and the contact radius, which compared well with the analytical model and with the Fletcher *et al.* [5] experimental results.

In 1993, Song *et al.* [7] studied the contact area of a bolted joint interface using analytical, finite element and experimental methods. They used the equation obtained by Chandrashekhara and Muthanna [8] for the contact stress distribution. Polycarbonate plastic samples were used to visualize the contact radius, while the contact stress was measured by a load cell. Continuing their work, Song *et al.* [9] developed a FEM code and an approximate model for the determination of the overall thermal resistance of bolted joints, formed by two oxygen-free copper flat smooth plates fastened by a stainless steel bolt. Their study was concentrated on the constriction resistance within the plates, since the contact resistance was neglected. They considered different boundary conditions from the Fletcher *et al.* [5] work. They concluded that the contact resistance depends on: the overlapping plate dimensions; the plate thicknesses; the thermal conductivities of the materials; and the size of the bolt head. A theoretical model for the Song *et al.* [9] correlation was proposed by Lee *et al.* [10].

As will be shown in this paper, the Fletcher *et al.* [5] and Lee *et al.* [6] correlations, both based on Fletcher *et al.* [5] data, and the model developed by Song *et al.* [9] are compared with the compact model developed in the present work, for the no washers between plates case. As the Fletcher *et al.* [5] boundary conditions are similar to the ones considered in this paper, the correlations are expected to compare better with the compact model.

None of the works available in the literature considered bolted joints with washers between plates. The influence of the temperature on the overall thermal resistance of bolted joints is also not addressed. Actually, Clausing [11], in 1966, justified the directional effect on the thermal contact resistances, using the thermal stress concept, but no model was developed.

RESISTANCE NETWORK

The analogy between thermal and electrical circuits is used to model the overall thermal resistance of bolted joints. Heat is supplied to the upper plate and dissipated from the lower plate (see Fig. 1). Within the plate constriction area, close to the washer, the heat follows one of two paths: either through the washers between the plates or through the bolt. Note that in Figs 1 and 2, the subscript w_0 refers to the washer between the bolt head and the upper plate, w_1 to w_n to washers between the plates (from the top to the bottom) and w_{n+1} to the washer between the lower plate and the nut. Similarly, p_1 refers to the upper plate, p_2 to the lower plate, h to the bolt head, b to the bolt shaft and n to the nut. The resistances are denoted R_c for contact, R_m for material and R_{ct} for constriction. For example, the symbol $R_{cw_{i,i+1}}$ represents the contact resistance between the i and $i+1$ washers. Just one resistance of each kind is shown in Fig. 1, for illustration.

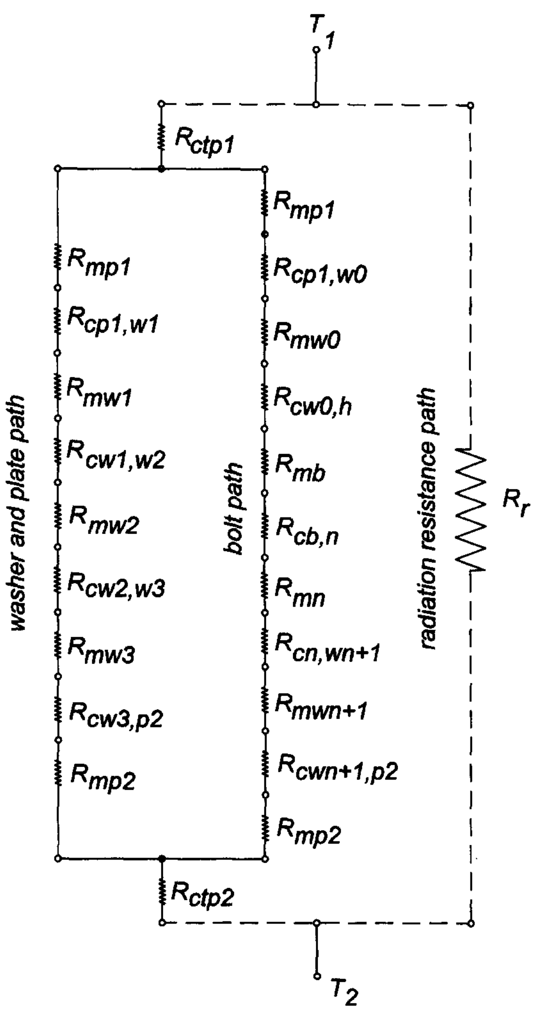

Fig. 2. Thermal resistance network for the bolted joint.

The washer network consists of the following resistances in series: upper plate material (R_{mp_1}), contact between the upper plate and washer (R_{cp_1,w_1}), washer material ($R_{mw}i$), contact between washers ($R_{cw_{i,i+1}}$), contact between the washer and lower plate (R_{cwn,p_2}) and lower plate material (R_{mp_2}). The bolt path consists of the following resistances in series: upper plate material (R_{mp_1}), contact between the plate and the bolt head washer (R_{cp_1,w_0}) washer material (R_{mw_0}), contact between the washer and the bolt head ($R_{cw_0,h}$), bolt material (R_{mb}), contact between the bolt and the nut (R_{cbn}), nut material (R_{mn}), contact between the nut and the washer ($R_{cn,w_{n+1}}$), washer material ($R_{mw_{n+1}}$), contact between the washer and lower plate (R_{cw_{n+1},p_2}) and lower plate material (R_{mp_2}).

The washer and bolt thermal resistance networks are thermally connected in parallel. This equivalent resistance is connected in series with the constriction resistances of the upper (R_{ct,p_1}) and lower (R_{ct,p_2}) plates. Figure 2 presents the overall resistance network. One should note that the number of contact resistances between washers and the number of washer material resistances increases with the number of washers. A third parallel path, the radiation resistance, is shown by dashed lines in Fig. 2, since, in most applications, its value is much larger than the other parallel resistances and it can be removed from the resistance network.

MATERIAL RESISTANCES

Figure 3 presents the physical model adopted to the formulation of the material resistances. For the washers and bolt shaft, the heat is supplied in the upper face and removed from the lower face. Therefore, the heat flux is one-dimensional and the thermal resistance can be determined by the simple expression:

$$R_m = \frac{t}{kA} \tag{1}$$

where t is the heat path length in the axial direction (bolt shaft length or washer thickness), k is the thermal conductivity of the material and A is the heat conduction area. In the washers w_0 and w_{n+1} the heat flux also experiences some constriction, but the material resistances R_{mw_0} and $R_{mw_{n+1}}$ (see Fig. 2) are estimated using eqn (1). The errors due to these approximations are negligible.

The upper and lower plate heat fluxes have a radial component. For the upper plate, heat is considered supplied uniformly over an area between the plate's

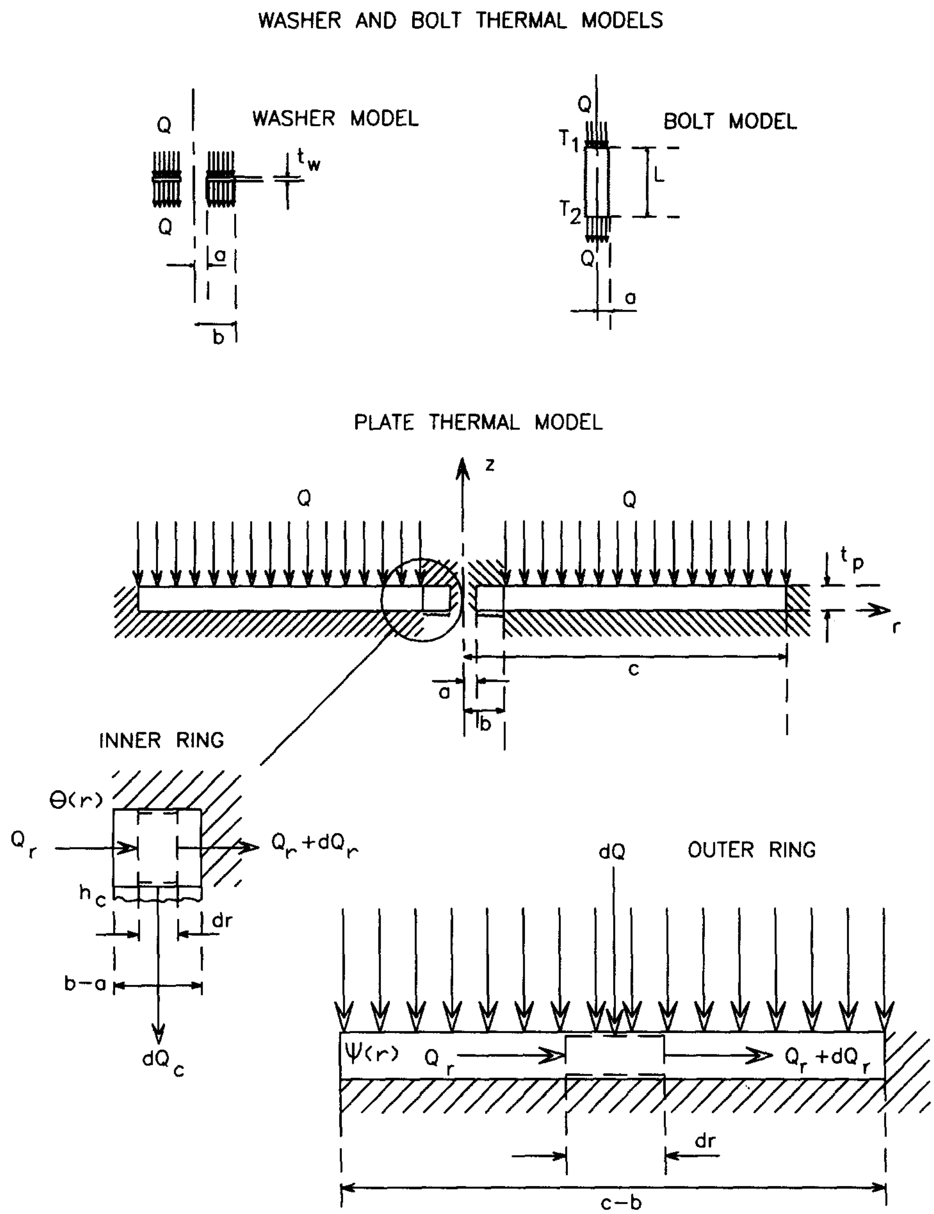

Fig. 3. Physical models for the material (washer, bolt and constriction) resistances.

external diameter and the external diameter of the washer between upper plate and the bolt head. This heat is removed also uniformly from its lower face over the area between the inside and outside diameters of the washer between plates. The same heat flux constriction occurs for the lower plate but with a mirror image.

The Biot number ($Bi = ht/k$, where h is the contact coefficient of heat transfer) is $\ll 0.2$. According to Yovanovich [12], the temperature distribution of the plates, which presents this condition, can be considered uniform in the thickness, or unidimensional in the radial direction.

Since this problem involves mixed boundary conditions (see Fig. 3), the plate is split into two rings: the internal ring, where $a \leqslant r \leqslant b$ and the external ring, where $b \leqslant r \leqslant c$. The dimension a corresponds to the diameter of the plate hole, b to the external diameter of the washer and c to the external diameter of the plate.

The inner ring temperature is defined as $\theta = T - T_s$, where T_s is the temperature of the surface that is contacting its lower face. The contact conductance between the inner, lower face and its contacting surface is h_c. The upper surface and the inner surface (radius a) are assumed adiabatic (see Fig. 3). Similarly, the outer ring temperature is defined as $\psi = T - T_s$. A known amount of heat flow rate Q is supplied to the upper surface of the plate. The lower surface and the outside surface, in the radial direction, are assumed adiabatic.

Mantelli *et al.* [13] showed that, for normal conditions, the contact resistance is small when compared to the overall thermal resistance. They also showed that the overall thermal resistance difference, using the plastic or elastic contact resistance models, available in the literature, is negligible. In this work, h_c is estimated through the Yovanovich [14] correlation.

Laplace's equation ($\nabla^2 T = 0$) is used, with the following boundary conditions:

$$1: \; r = a, \quad \frac{d\theta}{dr} = 0$$

$$2: \; r = c, \quad \frac{d\psi}{dr} = 0$$

$$3: \; r = b, \quad \theta = \psi$$

$$4: \; r = b, \quad \frac{d\theta}{dr} = \frac{d\psi}{dr} \tag{2}$$

Figure 3 shows the physical model used for the outer ring and the control volume over which the heat balance is made. The heat balance equation is:

$$Q_r + dQ = Q_r + dQ_r \tag{3}$$

where Q_r is the heat conducted in the radial direction ($Q_r = -kA(r)(d\psi/dr)$) and Q is the heat supplied over the upper area of the outer ring. The conduction area is $A(r) = 2\pi rt$, where t is the thickness of the plate. Substituting the appropriate expressions to eqn (3), one has:

$$\frac{d^2\psi}{dr^2} + \frac{1}{r}\frac{d\psi}{dr} + \frac{q}{kt} = 0 \tag{4}$$

The solution of eqn (4), which is the equation of the temperature distribution over the outer ring, is:

$$\psi = -\frac{q_i}{4kt} r^2 + C_1 \ln r + C_2 \tag{5}$$

Figure 3 also shows the control volume of the inner ring over which the heat balance is made. The heat balance equation is:

$$Q_r = Q_r + dQ_r + dQ_c \tag{6}$$

Q_r is determined as for eqn (3). The heat transferred through the contact area dA_c ($dA_c = 2\pi r\,dr$) is given by $dQ_c = h_e 2\pi r\,dr\theta$.

Substituting all the heat flow expressions into eqn (6), one obtains the following modified Bessel equation:

$$\frac{d^2\theta}{dr} + \frac{1}{r}\frac{d\theta}{dr} - \frac{h_c}{kt}\theta = 0 \tag{7}$$

The solution of the previous equation is:

$$\theta = C_3 I_0(mr) + C_4 K_0(mr) \tag{8}$$

with the parameter $m^2 = h_c/kt$.

Solving eqns (5) and (8) for C_1, C_2, C_3 and C_4, subjected to the boundary conditions (eqns (2)) one gets:

$$C_1 = \frac{qc^2}{2kt} \tag{9}$$

$$C_2 = \frac{\frac{q}{2kt}\left(\frac{c^2}{b} - b\right)}{m\left(\frac{K_1(ma)I_1(mb)}{I_1(ma)} - K_1(mb)\right)} \cdot \left(K_0(mb) - \frac{K_1(ma)I_0(mb)}{I_1(ma)}\right) + \frac{q}{2kt}\left(\frac{b^2}{2} - c^2 \ln b\right) \tag{10}$$

$$C_3 = \frac{-\frac{q}{2kt}\left(\frac{c^2}{b} - b\right)\frac{K_1(ma)}{I_1(ma)}}{m\left(\frac{K_1(ma)I_1(mb)}{I_1(ma)} - K_1(mb)\right)} \tag{11}$$

$$C_4 = \frac{\frac{q}{2kt}\left(\frac{c^2}{b} - b\right)}{m\left(\frac{K_1(ma)I_1(mb)}{I_1(ma)} - K_1(mb)\right)} \tag{12}$$

The thermal constriction resistance is defined as:

$$R = \frac{\bar{\psi} - \bar{\theta}}{Q} \tag{13}$$

where $\bar{\psi}$ and $\bar{\theta}$ are the outer and inner ring mean temperatures, respectively. These temperatures are given by:

$$\bar{\psi} = \frac{2}{(c^2 - b^2)} \int_b^c \psi(r) r\,dr \tag{14}$$

and

$$\bar{\theta} = \frac{2}{(b^2 - a^2)} \int_a^b \theta(r) r\,dr \tag{15}$$

One observes that, according to the boundary conditions, this model includes the plate material (R_{mp}) and the contact ($R_{cp,w}$) resistances.

RADIATION RESISTANCE

The radiation resistance is defined as the ratio between the temperature difference of two surfaces that radiate to each other and the net transferred heat. If the temperature difference between the surfaces is not large, the heat transferred by radiation is small when compared to the other two models, conduction and convection. For satellite applications (vacuum environment), the radiation is important for external surfaces which face the Sun or space. This importance decreases for the interior compartments of the satellite. To estimate the radiation resistance, the model developed in this paper can be used.

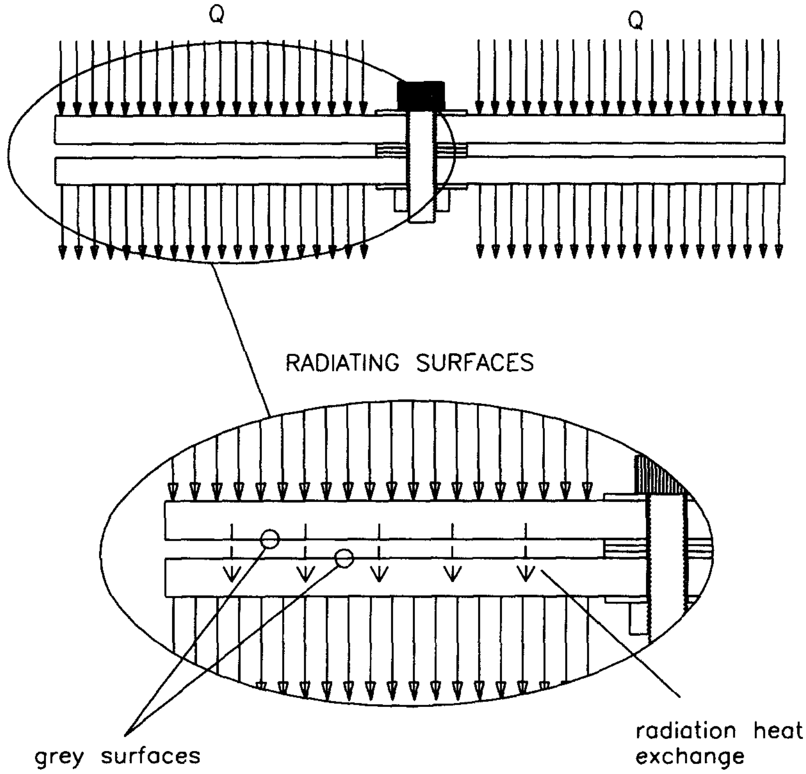

Fig. 4. Physical model for the radiation resistance between plates.

Radiation heat transfer is assumed between the lower face of the upper plate and the upper face of the lower plate, as shown in Fig. 4. The radiation resistance can be determined from the enclosure formed by two infinite parallel flat grey surfaces, whose temperatures are known. This simplification is considered reasonable since the spacing between the plates is small when compared to the diameter of the disks. If the temperature difference between the surface is small, the radiation heat transfer coefficient h_r can be approximated by [12]

$$h_r = \frac{4\sigma T_m^3}{1/\varepsilon_{p_1} + 1/\varepsilon_{p_2} - 1}$$

where T_m is the average temperature between the upper and lower plates and ε_{p_1} and ε_{p_2} are the emissivities of the internal faces of plate 1 (upper) and 2 (lower), respectively. The radiation resistance for the heat exchange area A_p is given by:

$$R_r = \frac{1}{h_r A_p} = \frac{\dfrac{1}{\varepsilon_{p_1}} + \dfrac{1}{\varepsilon_{p_2}} - 1}{4\sigma T_m^3 \pi (c^2 - b^2)} \qquad (16)$$

THERMAL STRESS ANALYSIS

The contact stress between the surfaces is an input parameter for the calculation of the contact conductance (h_c). Several studies [8, 15–17] have examined the pressure distribution between two contacting surfaces of plates fastened by a bolt. These have shown that the stress at near-bolt regions is large and it diminishes radially until it vanishes. None of these models considered joints assembled with washers between the plates. In the present study, the dimensions of the washers are much smaller than the dimensions of the plates. Fontenot [3] observed that the stress between the inside surface of a flat bolt head and a flat surface can be considered uniform. Based on his observations, the stress distribution between contacting surfaces is considered uniform over the contact area.

A decrease in the initial contact pressure can occur when the bolted joints are assembled at room temperature and submitted to low temperature levels, as observed by Mantelli and Yovanovich [18] and Mantelli and Basto [19]. This behavior depends on the coefficients of thermal expansion and modulus of elasticity of the bolt, washers and plates. A model to calculate the contact stress between bolted joint components as a function of the temperature is developed based on thermal stress theory.

Gatewood [20] solved a similar problem for bolted joints with the forces acting perpendicularly to the centerline of the bolt shaft. In the present paper, the forces are assumed to act parallel to the shaft. A simple physical model for the thermal stress analysis of bolted joints is used, as sketched in Fig. 5. The radial and angular stress components are considered null. The displacement of any element in the z-direction, due to the applied stress and to the temperature variation, is [20] $w = (P/E)L + \alpha \Delta T L$, where $\Delta T = \bar{T} - T_0$, is the

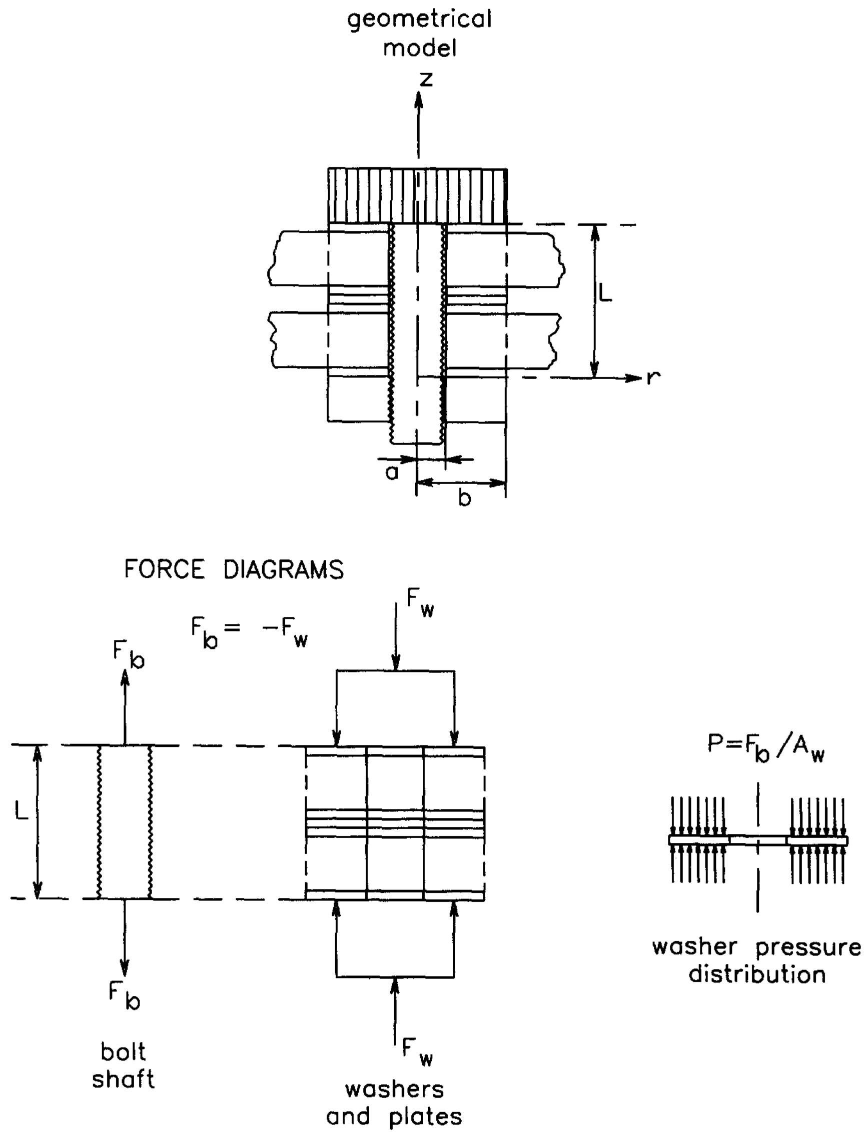

Fig. 5. Physical model for the thermal stress formulation.

difference between the final and room temperatures, P is the applied stress, L is the length of the material under stress, E is the modulus of elasticity of the material and α is the coefficient of thermal expansion of the material. The α and E subscripts ss and al refer to stainless steel and aluminum materials, respectively. By convention, a tensile stress is positive and a compressive stress is negative. In this study the bolt stress P_b is positive and the washers and plates stresses P_w are negative.

The overall deformation of the bolted joint can be expressed as the combination of the deformation of each element. An additional assumption is made to simplify the final equation: all the physical properties used are obtained for the bolted joint average temperature, $\bar{T}$. This simplification is reasonable for this problem, where both plates are of the same material, have similar dimensions and are in physical contact with identical washers. For more complex systems, each component would require a separate study. The deformation of the bolt shaft is then compared with the sum of the individual deformations of the washers and plates (compatibility condition). From the force balance, $\Delta P_w A_w = \Delta P_b A_b$, one has:

$$\Delta P_w = \frac{2L_p E_{ss}(\alpha_{al}-\alpha_{ss})(\bar{T}-T_0)}{\left[(n+2)L_w\left(1-\frac{(b^2-a^2)}{r_b^2}\right)-2\frac{(b^2-a^2)}{r_b^2}L_p+2L_p\frac{E_{ss}}{E_{al}}\right]} \tag{17}$$

where A_w is the washer area, A_p is the bolt shaft area, n is the number of washers between the plates, L_p is the thickness of the plates, L_w is the thickness of the washers and r_b is the radius of the bolt shaft. The final washer stress is $P_f = P_t + \Delta P_w$, where P_i is the initial stress. Equation (17) shows that, if the operating temperature level of the satellite, $\bar{T}$, is smaller than assembling temperature, T_0 then ΔP_w is negative and, in the limit, the final stress can reach the zero value. For low contact stress cases, the importance of the contact resistance increases and it may dominate the overall resistance. When the contact stress vanishes, the surfaces lose their contact and the radiation turns out to be the only heat transfer means between plates. The final stress (P_f) is used as input parameter for the calculation of the contact conductance h_c, which, in turn, is used as the input parameter for the calculation of the constriction resistance.

COMPACT MODEL

Figure 2 shows that many different thermal resistances, which depend on many different parameters, are necessary to determine the overall thermal resistance of bolted joints. With the objective of simplifying the model, Mantelli [21] developed a parametric study to find which parameters have the major influence and which resistance paths can be omitted from the circuit. From this analysis and for normal levels of contact stresses, the bolt resistance and the radiation paths were eliminated from the circuit, since they present thermal resistances that are much larger than the washer and plates' path. After the similar resistances are grouped, the resistance network, presented in Fig. 2, is reduced to three different types of resistances connected in series. These are the constriction resistance, which includes the plate material and contact between plate and washer (or between plates, for the no washers between plate case), the washer material resistance and the contact resistance between washers.

Mantelli [21] verified that $\bar{\theta}$ is much smaller than $\bar{\psi}$. Therefore $\bar{\theta}$ can be neglected in eqn (13), if $a/b < 0.8$ and $c/b < 180$, resulting in:

$$R_{ct} = \frac{\bar{\psi}}{Q} \tag{18}$$

The following compact version of the constriction resistance is valid for $b/c < 0.3$:

$$R_{ct} = \frac{1}{2\pi k_p L_p}\left[\ln\left(\frac{c}{b}\right) - \frac{3}{4} + \Phi\right] \tag{19}$$

where Φ is given by:

$$\Phi = \frac{\dfrac{K_0(\lambda)}{K_1\left(\lambda\dfrac{a}{b}\right)} + \dfrac{I_0(\lambda)}{I_1\left(\lambda\dfrac{a}{b}\right)}}{\lambda\left[\dfrac{I_1(\lambda)}{I_1\left(\lambda\dfrac{a}{b}\right)} - \dfrac{K_1(\lambda)}{K_1\left(\lambda\dfrac{a}{b}\right)}\right]} \tag{20}$$

and the λ parameter is:

$$\lambda = \sqrt{\frac{h_c b^2}{k_p L_p}} \tag{21}$$

Also, according to Mantelli [21], Φ is 0.1 for most of the practical cases ($a/b < 0.8$ and $\lambda > 3.0$).

Applying eqns (1) and (19), and Yovanovich's [14] correlation for contact conductance, to a reduced resistance network, one has:

$$R = 2R_{ct} + n\frac{L_w}{k_w \pi (b^2 - a^2)} + (n-1)\frac{\dfrac{\sigma}{m}}{\pi(b^2-a^2)k_s\left(\dfrac{P_f}{H}\right)^{0.95}} \tag{22}$$

valid for $n \geqslant 1$, where σ/m is the RMS roughness of the contacting surfaces over the average slope, k_s is the harmonic mean conductivity of the materials of the two contacting surfaces and H is the hardness of the softer material. The constriction resistance (R_{ct}) is multiplied by two because it appears twice in the network (two plates). For $n = 0$, this equation reduces to the first term, or to the constriction resistance only. This compact version of the model will be used in the comparison with data and the literature models in the next sections.

The relative importance of the resistances in a bolted joint can be observed from Fig. 6, which presents theoretical plots for the one washer between plate case ($n = 1$). The complete analytical model (no simplifications) is plotted with a solid line. The compact model, eqn (22), is plotted with a short-dashed line. The compact and complete model curves are so close to each other (maximum difference of about 0.2%) that it is difficult to identify them. Therefore, due to its simplicity, the compact model is used in this paper as the analytical model.

Figure 6 also presents two more plots, the contact resistance (long dashed line) and the compact model for no contact resistance (or $\lambda = 0$ and $\Phi = 0$). These two curves are the asymptotes of the analytical model. At low temperature levels, the contact resistance curve is very close to the analytical model, showing that the contact resistance is dominant. For high temperature

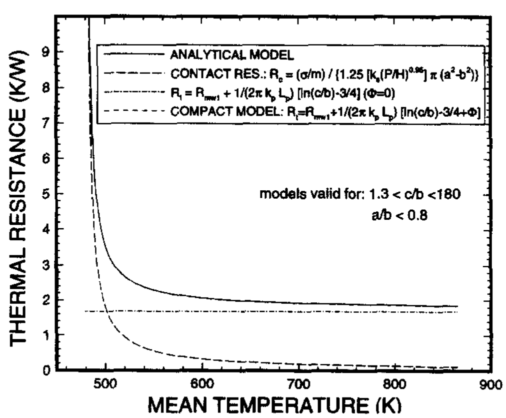

Fig. 6. Comparison of the entire and compact analytical models, showing their asymptotes.

levels, the contact resistance is a small fraction of the overall thermal resistance and the model approximates to the compact model for no contact resistance. For this specific bolted joint and for the high temperature levels, the contact resistance represents about 11%, the washer material resistance represents 56% and the constriction resistance represents 33% of the total resistance.

COMPARISON OF THE PRESENT MODEL WITH LITERATURE MODELS AND DATA

In this section, the present model is compared with Fletcher *et al.* [5] data and correlations. Also, the model is compared to the constriction resistance model developed by Song *et al.* [9]. As observed before, these comparisons are possible only for the no washer between plate case.

The first comparison is shown in Fig. 7, where a plot of the non-dimensional resistance R^* as a function of the contact radius b is presented, for one case studied by Fletcher *et al.* [5]. They defined in the non-dimensional resistance as $R^* = (R_{\text{total}} - R_{\text{bulk}})/R_{\text{bulk}}$, where R_{bulk} is the total resistance (R_{total}) for uniform flow within the plate. In their work, the heat flux was modeled as uniformly supplied over the upper face of the upper plate, while, in the present paper, it is limited to the area between the outer washer and outer plate radii. This difference is not important if the washer dimensions are small when compared to the plate dimensions. The Lee *et al.* [6] correlation, based on the Fletcher *et al.* [5] study, is also plotted in Fig. 7. The present model and the correlations are expected to compare well, as it is observed in this figure.

When there is at least one washer between the plates, the apparent contact area is well defined (equal to the washer area). If the washers are absent, the contact area is expected to be larger (see Song *et al.* [7]). For the present work, the contact area was assumed to be identical to the washer, even for the no washers between plate case. This simplification does not affect the comparison presented in Fig. 7, since the contact area is an input parameter for the compact model and correlations.

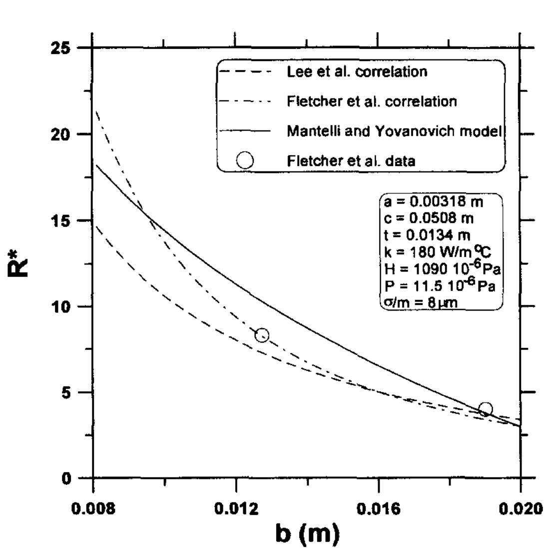

Fig. 7. Comparison between Fletcher *et al.* [5] data, correlations and the present compact model.

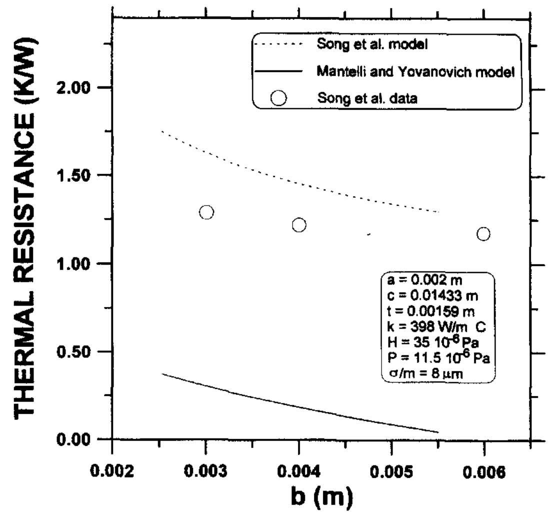

Fig. 8. Comparison between the model and data of Song *et al.* [9] and the present compact model.

In Fig. 8, the compact and the Song *et al.* [9] models are compared. Dimensional resistances are used because the bulk resistance concept adopted by Song *et al.* [9], defined in the radial direction, is different from the Fletcher *et al.* [5] concept, which is in the axial direction.

Song *et al.* [9] included in their constriction model the calculation of the contact area, based on the model developed by Song *et al.* [7]. To enable the present comparison, the Song *et al.* [7] model was used for the determination of the contact area, an input parameter for the present compact model. Since the boundary conditions adopted for both cases are very different, the models do not compare well, as expected. The data obtained by these authors compare better with the Song *et al.* [9] model, since their experiment was designed to reproduce their model. Therefore, Fig. 8 shows that it is not possible to use the Song *et al.* [9] model for boundary conditions similar to the ones adopted in the present paper.

COMPARISON OF MODEL AND DATA

Mantelli and Yovanovich [22] compared the model with data obtained from a experimental parametric work, based on satellite bolted joints. Figure 9 reproduces the overall thermal resistance as a function of the mean temperature for mountings with three washers between plates (n = 3). The theoretical curve was obtained using eqn (22). In this plot, the influence of the temperature on the overall thermal resistance of experimental and theoretical results is evident. The aperture torque, applied to the experimental mount-

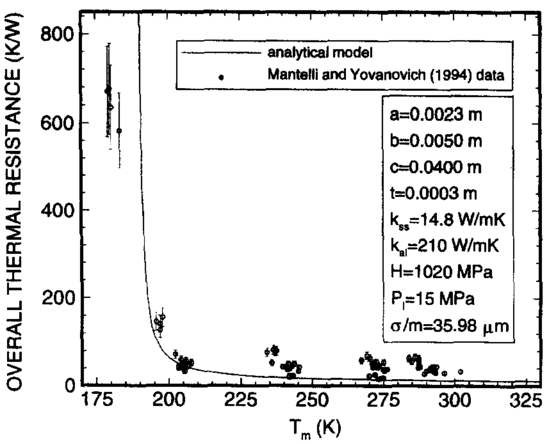

Fig. 9. Comparison between Mantelli and Yovanovich [18] data and the compact model for the three washer mountings.

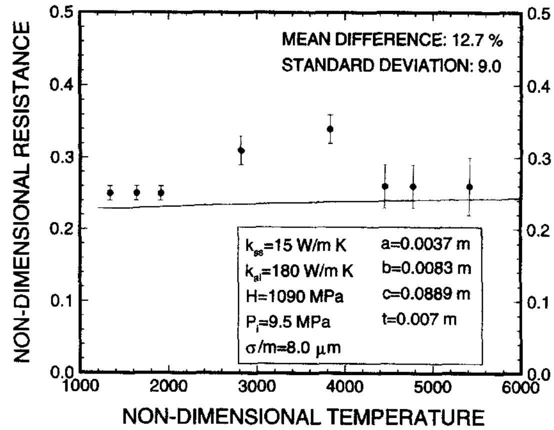

Fig. 10. Comparison between Mantelli [21] data and the present compact model, for one washer mounting.

ings during their assembly, was measured. The initial stress was obtained by comparison of the data with the theoretical curves, as shown in Mantelli and Yovanovich [22]. The comparison between the data and model is good, especially for low temperature levels. The vertical bars represent the experimental uncertainty of the overall thermal resistance data. As these data were obtained from the measurement of experimental mountings whose geometrical and physical properties were not well controlled, a large range of variation among similar mountings was found, for high temperature levels. This range of variation was discussed by Mantelli and Yovanovich [22].

To obtain a better comparison with the model, Mantelli [21] developed another set of experiments, for the one washer between plate configuration, where the physical properties of the mounting components were well controlled. Therefore, the range of variation among similar mountings, observed in Fig. 9, was eliminated. The non-dimensional resistance is defined as $R^* = Rk_sL_s$, where k_s and L_s are the harmonic mean value of the thermal conductivity and the thickness ($k_s = 2k_pk_w/(k_p+k_w)$ and $L_s = 2L_pL_w/(L_p+L_w)$, respectively). This non-dimensional resistance is plotted against the non-dimensional temperature ($T^* = T_mk_sL_s/(qb^2)$) and compared to the theoretical curves, as presented in Fig. 10. For this case, the contacting stress was monitored by a washer shaped load cell, which substituted for one of the washers of the mounting (see Mantelli [21]). From this figure, one observes that the comparison between the data and model is excellent (around 13%), especially if one considers the complexity of the system. The bars shown in Fig. 10 represent the experimental uncertainty for each point. The reader should refer to Mantelli [21] for a complete analysis, presentation and discussion of the experimental work and data.

SUMMARY AND CONCLUSIONS

The main objective of this work was to develop an analytical model to predict the overall thermal resistance of bolted joints in a vacuum environment, especially for satellite applications. A simple and compact model, where the input data are the geometric and the physical parameters, was obtained. A model to predict the contact stress between the boiled joint elements as a function of the temperature was also developed.

First, a thermal resistance network was introduced. Included in the network were: material resistances; contact resistances; and radiation resistance. No convection heat transfer was expected (the bolted joint was in a vacuum environment). A model, from current literature, was used to calculate the contact conductance. In most applications, the radiation resistance is large and can be eliminated from the thermal network. In the cases analyzed in this paper, both the radiation and the bolt path resistances were eliminated from the network. Algebraic simplifications were applied and a compact model was obtained. The model was compared with other analytical models, correlations and data available in the literature, for the no washers case. This comparison was very good for the cases where the adopted boundary conditions were similar to the ones adopted in this paper. The model was also compared with data for the three washers between plate configuration. In this comparison, the dependence of the overall thermal resistance of the temperature was verified. Finally, the model was compared with data for the one washer between plate configuration. From this comparison, the precision of the model was determined. Concluding, the model is strongly recommended for the calculation of the overall thermal resistance of bolted joints due to its precision and simplicity (it can be carried out on a hand calculator). The conductivities and the dimensions of the materials are the only necessary input data.

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